Modelling Heterogeneity

in the Measurement of Structural Efficiency

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1. Introduction

Researchers have continually attempted to derive a vigorous method for measuring the overall productive efficiency of a group of production units. Farrell (1957) is the first to propose the concept of "structural efficiency" to measure the overall efficiency of an industry (as a group of firms). He suggested that it can be measured by the weighted average (by output) of the efficiency scores of individual units. Førsund and Hjalmarsson (1979) contended that the structural efficiency could be better measured by the efficiency of the average unit of a group. However, Ylvinger (2000) argued that these two approaches might lead to confusing and puzzling results. He proposed various weighting methods that could overcome the problems. However, no theoretical reason has been given for justifying either one of the weighting systems. Another approach is proposed by Färe, Grosskopf, and Li (1992). In a framework of single output and multiple inputs, they derived group efficiency measures that takes into account whether the inputs are reallocatable across production units. Their study, however, does not provide any instrument for handling the case of multiple output. Generalizing the measure of Färe, Grosskopf and Li (1992), Li and Ng (1995) developed a framework for measuring the efficiency of a group with multiple outputs and multiple inputs. Their method utilises the concept of shadow price that facilitates the computation and decomposition of group efficiency in a multiple output framework. However, they have not spelt out the implications of this approach. More recently, Li and Cheng (2005a) laid out the framework in a more general fashion and spelt out the relationship between Li and Ng (1995) and other approaches. Li and Cheng (2005b) further pointed out that the model of Li and Ng (1995) can solve the puzzles related to the measuring method of structural efficiency in the literature.

Despite these developments, all the existing methods of measuring structural efficiency assume that the group of production units utilizes an identical technology. This is not an uncommon assumption in productivity analysis. In fact, most analyses of technical efficiency (for individual production units) have implicitly assumed. However, technology is not always the same, particularly when there are obvious heterogeneities across production units. There are reasons for the existence of subgroups with some heterogeneity. The first is heterogeneous technologies, which means that some technical know-hows are not available to all firms or firms intentionally do not consider some production techniques. Mills and Schumann (1985) showed that, with demand fluctuations, it was possible that technologically diverse firms competing with each other if there is a tradeoff between static-efficiency and flexibility. The second is an institutional factor. Different ownerships of enterprises, for instance, may face very different policy environments. They might have access to the same technology but their behaviour might be constrained by policies imposed on them by the government. In the case of China, for instance, state-owned enterprises have a lot of policy burdens and institutional constraints (Lin and Tan, 1999). They may also suffered from various incentive problems due to its rigid management structure. We expect each subgroup (as classified by ownership) of enterprises behave differently at least in a fairly long period of time, even if they face the same technology. Hence we will consider the efficiency measures of a production group in which subgroups of firms with different behvaior.

This paper suggests and compares several possible strategies to model heterogeneity in production units when measuring the structural efficiency of a production group. Classifying the production units into different groups and treating them separately is a simple but probably not the best solution. We consider two other modeling strategies. The first one is to include environmental factors as inputs of the production function. The underlying assumption is that the production units share the same basic production technology that determines the relationship between (traditional) inputs and outputs. The second one is to assume that production units have different production technologies and construct a group frontier based on these units. We will introduced a new extended frontier based on the putty-clay model initiated by Johansen (1970) and Koopmans (1977) and applied to efficiency analysis by Färe and Primont (1984, 1993). To do this, we have to overcome some difficulties in specifying the theoretical production technologies and making them empirically applicable.

This paper is organized as follows. Section 2 discusses the measure of structural efficiency for a group of production units. Section 3 specifies the production technology and the structural efficiency measure when environmental factors are considered. Section 4 develops methodologies for modelling hereogeneous technologies. Section 5 presents the empirical applications of the proposed methodologies to Chinese agriculture, utilizing county-level data. Section 6 concludes.

2. The Measure of Structural Efficiency

To facilitate further discussion, we consider K firms in an industry. "Firm" is any basic production unit and "industry" consists of a group of production units. Each firm produces M outputs by utilizing N inputs. The kth firm produces an output vector $y^k \in \mathbb{R}_+^M$ by empolying input vector $x^k \in \mathbb{R}_+^N$, $k = 1, \cdots, K$. The production technology of firm k is defined by the technology set $T^k = \left\{(x, y) : x \text{ can produce } y\right\}$.

Previous confusions about structural efficiency is that researchers ignored that structural efficiency is a measure of the performance of industry. Technical efficiency is a measure of the performance of a firm. As pointed out by Li and Cheng (2005b), some confusions come from the fact that the performance of an industry cannot be found just by investigating the firm frontiers. To measure the structural efficiency of an industry, we need to formulate the aggregate frontier of the industry. Given individual technology set T, the aggregate technology set of the industry is

$$\hat{\mathfrak{I}} \tag{1}$$

$$= \{(X, Y)\ :\ Y \text{ can be produced by the } J \text{ firms together as a group, given total endowment } X\}$$

$$= \left\{(X, Y)\ :\ (x^k, y^k)\ \in\ T^k,\ k\ =\ 1, \cdots, K;\ \sum_{k=1}^{K}(x^k, y^k)\ =\ (X, Y)\right\}$$

$$= \sum_{k=1}^{K} T^k.$$ [1]

This aggregate technology set describes all feasible aggregate input-output combinations of the K firms as a whole. Once the aggregate technology set is known, the structural efficiency of an industry is obvious.

Let the observed input and output vectors of individual production units be (x_k^o, y_k^o), $k\ =\ 1, \cdots, K$. Then the aggregate input and output vectors of the whole industry are $(X^o, Y^o)\ =\ \sum_{j=1}^{J}(x_j^o, y_j^o)$. Similar to the framework of Li and Ng (1995) and using the terminologies of Li and Cheng (2005), the *output-oriented structural efficiency measure*[2] of the group is

$$H(X^o, Y^o)\ =\ \max_{\theta}\left\{\theta\ :\ (X^o, \theta Y^o)\ \in\ \hat{\mathfrak{I}}\right\}. \tag{2}$$

In other words, the structural efficiency measure is to find the maximum proportional expansion of the aggregate output vector, given the input endowment of the industry. By definition, $H\ \geqq\ 1$. The whole group is structurally efficient if $H\ =\ 1$. When $H\ =\ 1.12$, say, this means that the current group outputs can be raised proportionally by 12% without adding any new resources.

3. Measuring Structural Efficiency with Uncontrollable Factors

Suppose one is willing to believe that all firms are facing the same technology and, at the same time, wants to take into account heterogenity among firms. How can he achieve this goal? This is possible when there are some uncontrollable factors that may constrain the performance of a firm using the on-going technology.

We assume that all firms face the same technology but each of them is unique in some sense. Suppose the kth firm produces an output vector $y^k \in \mathbb{R}_+^M$ by employing input vector $x^k \in \mathbb{R}_+^N$, $k = 1, \cdots, K$. In the technology set, there is an uncontrollable vector $s \in \mathbb{R}_+^J$ which reflects the heterogeneity among firms. Thus the vector s can affect the production relation between y^k and x^k. The production technology of firm k is defined by the technology set $T = \left\{(x, s^k, y) : (x, s^k) \text{ can produce } y\right\}$. Since the vector s^k cannot be controlled by the firm, it is like a fixed input vector. Thus the actual technology set faced by firm k for given s^k is $T(s^k) = \{(x, y) : (x, s^k, y) \in T\}$. Replacing T^k in (1) by $T(s^k)$, we get the aggregate technology set of the industry:

$$\hat{\mathfrak{I}}(s^1, \cdots, s^K) = \left\{(X, Y) : (x^k, s^k, y^k) \in T(s^k), k = 1, \cdots, K; \sum_{k=1}^{K}(x^k, y^k) = (X, Y)\right\} \quad (3)$$

$$= \sum_{k=1}^{K} \mathfrak{I}(s^g).$$ [1]

This aggregate technology set describes all feasible aggregate input-output combinations of the K firms as a whole for a given allocation of the uncontrollable factor $(s^1, \cdots, s^K)$. Replacing $\hat{\mathfrak{I}}$ in (2) by $\hat{\mathfrak{I}}(s^1, \cdots, s^K)$, the structural efficiency measure of the industry for the given allocation of the uncontrollable factor is

$$H\Big(X^o, Y^o; s^1, \cdots, s^K\Big) = \max_\theta\Big\{\theta : (X^o, \theta Y^o) \in \widehat{\Im}(s^1, \cdots, s^K)\Big\}$$
$$= \max_\theta\left\{\theta : (X^o, \theta Y^o) \in \sum_{k=1}^{K} T(s^g)\right\}$$
$$= \max_\theta\left\{\theta : (x^k, s^k, y^k) \in T(s^k), k = 1, \cdots, K;\ \sum_{k=1}^{K}(x^k, y^k) = (X^o, \theta Y^o)\right\}$$

Proposition (1) *Let the firm technology set, $T = \Big\{(x, s, y) : (x, s)$ can produce $y\Big\}$, be a convex cone. If $s \in \mathbb{R}_+^J$ is an uncontrollable factor and $s^k = \alpha_k s^1$, $k = 1, \cdots, K$, then $H\Big(X^o, Y^o; s^1, \cdots, s^K\Big) = H(X^o, Y^o)$.*

Proof: Let $\alpha^* = \sum_{k=1}^K \alpha_k$. Since T is a convex cone, $cT = T$ for any positive contant c and $T = \widehat{\Im}$. Then

$$\Im(s^k) = \{(x, y) : (x, s^k, y) \in \Im\} \tag{5}$$
$$= \left\{\alpha_k\left(\frac{x}{\alpha_k}, \frac{y}{\alpha_k}\right) : \alpha_k\left(\frac{x}{\alpha_k}, s^1, \frac{y}{\alpha_k}\right) \in \Im\right\}$$
$$= \alpha_k\{(x, y) : (x, s^1, y) \in \Im/\alpha_k = \Im\}$$
$$= \alpha_k \Im(s^1).$$

Let $S^o = \sum_{k=1}^K s^k = \sum_{k=1}^K \alpha_k s^1 = \alpha^* s^1$. It follows that the industry technology is

$$\widehat{\Im}(s^1, \cdots, s^K) = \sum_{k=1}^{K} T(s^k) = \sum_{k=1}^{K} \alpha_k T(s^1) = \left(\sum_{k=1}^{K} \alpha_k\right) T(s^1) = \alpha^* T(s^1). \tag{6}$$

$$H\Big(X^o, Y^o; s^1, \cdots, s^K\Big) = \max_\theta\Big\{\theta : (X^o, \theta Y^o) \in \widehat{\Im}(s^1, \cdots, s^K)\Big\} \tag{7}$$
$$= \max_\theta\{\theta : (X^o, \theta Y^o) \in \alpha^* T(s^1)\}$$
$$= \max_\theta\left\{\theta : \left(\frac{X^o}{\alpha^*}, \theta\frac{Y^o}{\alpha^*}\right) \in T(s^1)\right\}$$
$$= \max_\theta\left\{\theta : \left(\frac{X^o}{\alpha^*}, s^1, \theta\frac{Y^o}{\alpha^*}\right) \in T\right\}$$
$$= \max_\theta\{\theta : (X^o, \alpha^* s^1, \theta Y^o) \in \alpha^* T\}$$

Recall that $\alpha^* T = T$ under constant returns to scale and T is convex and $\widehat{\Im} = T$, we have

$$H\left(X^o, Y^o; s^1, \cdots, s^K\right) = \max_{\theta}\{\theta : (X^o, S^o, \theta Y^o) \in T\}$$
$$= H(X^o, Y^o).$$

Q.E.D.

Proposition (1) states that the structural efficiency will be the same whether an input subvector is controllable or not, provided that all observed subvectors are proportional to each other. Thus when there is only one uncontrollable input, the structural efficiency will be the same for identical firm technology exhibiting constant returns to scale.

4. Heterogeneous Technologies

Consider K observed firms: (x^k, y^k), $k = 1, \cdots, K$, each of which has a distinct technology. Since there is only one observation on each technology, it is impossible to study any efficiency issue given such limited data. To measure structural efficiency with heterogeneous technologies, we need more information for each technology. It is helpful to consider the a hierarchy of different production units. Suppose we may want to gauge the overall performance of an industry which consists of G firms. Each of the firms, in turn, consists of K_g plants, $g = 1, \cdots, G$. We have to consider whether we assume each firm or each plant to have a distinct technology. Likewise, while we apply the methods to regional data, we have to consider what level of regions we should analyze. In general, there are four mutually exclusive cases:

Case 1 (SS):	Same technology among firms	Same technology within firms
Case 2 (SD):	Same technology among firms	Different technologies within firms
Case 3 (DS):	Different technologies among firms	Same technology within firms
Case 4 (DD):	Different technologies among firms	Different technologies within firms

There is no heterogeneity in Case SS. Let $T^{gi} = \left\{(x, y) : x \text{ can produce } y\right\}$ be the technology of plant i in firm g. In Case SS, $T^{gi} = T$ for all g and i. Applying (1), the technology of firm g is $\mathfrak{I}^g = \sum_{i=1}^{K_g} T^{gi} = T$. If T is a convex set, then $\mathfrak{I}^g = K_g T$. The aggregate technology of industry is $\widehat{\mathfrak{I}} = \sum_{g=1}^{G} \mathfrak{I}^g = \sum_{g=1}^{G} K_g T = \left(\sum_{g=1}^{G} K_g\right) T$. Following Li and Ng (1995),

$$H(X^o, Y^o) = \max_{\theta}\left\{\theta : (X^o, \theta Y^o) \in \widehat{\mathfrak{I}}\right\} = \max_{\theta}\{\theta : (\bar{x}^o, \theta\bar{y}^o) \in T\} \tag{8}$$

where $\bar{x}^o = X^o / \left(\sum_{g=1}^{G} K_g\right)$ and $\bar{y}^o = Y^o / \left(\sum_{g=1}^{G} K_g\right)$. Only one technology set, the technology set of a typical plant, is necessary to find the structural efficiency. This is equivalent to the average firm measure suggested by Førsund and Hjalmarsson (1979). This is simple to find. Let the observed input and output vectors be x_k^o and y_k^o, $k = 1, \cdots, K$. If the plant technology is the VRS technology, then the structural efficiency is

$$\begin{aligned} H &= \max_{\theta, z} \theta \\ \text{subject to} &: \sum_{g=1}^{G}\sum_{i=1}^{K_g} z_{gi} y^{gi} \geqq \theta \bar{y}^o \\ &\sum_{g=1}^{G}\sum_{i=1}^{K_g} z_{gi} x^{gi} \leqq \bar{x}^o \\ &\sum_{g=1}^{G}\sum_{i=1}^{K_g} z_{gi} = 1. \end{aligned} \tag{9}$$

Case SD seems not reasonable. When the technologies of plants are different within a firm, it is hard to imagine why the firms would share the same technology. Hence this case is not considered here.

Thus heterogeneity mainly appear in Cases DS and DD. There are two difficulties in the modelling of heterogeneity of firms. Firstly, as pointed out by Li and Cheng (2005b), the structural inefficiency consists of technical inefficiency and allocative inefficiency within firms and reallocation of inputs among firms. All firms must be considered in finding the structural inefficiency. Empirically, this may involve a huge programming problem for even a data set of relatively small size. If all the firms and plants are identical, then the structural efficiency measure is reduced to the Farrell technical efficiency measure of the average firm, as shown in (8). When there is heterogeneity, (8) is not applicable. Simplifying the computation is an important matter that affects the practicality of the model. Secondly, even if we do not care about the programming size problem, sometimes we have to consider heterogeneity down to the most basic units (plants in our case). However, in some empirical model, for example, the putty-clay model discussed in Johansen (1970) and Koopmans (1977), the industry is always structurally efficient. This is not an useful result. To make the model practical, we have to build in some new elements. Existing literature on efficiency analysis does not provide any answers to these two issues. In the following subsection, we suggests two possibilities for the Cases DS and DD.

4.1 Different Firm Technologies and Identical Plants within Firms

Let the technology of each plant in firm g be T^g. As mentioned just before (8), $\mathfrak{I}^g = K_g T^g$ for identical plant technology within firm g. The aggregate technology set for the whole group is $\widehat{\mathfrak{I}} = \sum_{g=1}^{G} \mathfrak{I}^g = \sum_{g=1}^{G} K_g T^g$. The stuctural efficiency of the industry is then

$$H(X^o, Y^o) = \max_\theta \left\{ \theta : (v^{gi}, u^{gi}) \in T^g, \, i = 1, \cdots, K_g, \, \sum_{g=1}^{G} \sum_{i=1}^{K_g} (v^{gi}, u^{gi}) = (X^o, \theta Y^o) \right\}.$$

Before we proceed to the structural efficiency, we note that structural efficiency requires increasing total output of the industry by moving inputs from some firms to others. When a reallocation takes place, the total endowment does not change. We will say any given $\left(x^{gi},\ i = 1,\cdots,K_g\right)$ for firm g is an allocation of X^g if $\sum_{i=1}^{K_g} x^{gi} = X^g$. An allocation is equal if $x^{gi} = X^g/K_g$ for all i. It is feasible for (X^g, Y^g) if it is an allocation of X^g and there exists y^{gi}, $(x^{gi}, y^{gi}) \in T^g$ for all i such that $\sum_{i=1}^{K_g} y^{gi} = Y^g$. An allocation which is feasible in one total endowment may not be feasible in another. However, a special allocation is feasible for any endowment of resources and the accompanying outputs when the technology is convex.

Proposition (2) *If the plant technology T^g is convex and identical for every plant in firm g, then an equal allocation of X ($x^{gi} = X/K_g$ for all i) is feasible for any $(X,Y) \in \Im^g$.*

Proof: Let $(x^{g1},\cdots,x^{gK_g})$ be a feasible allocation for (X,Y). This means $(x^{gi}, y^{gi}) \in T^g$ for all i for some $(y^{g1},\cdots,y^{gK_g})$. Since T^g is convex, $\sum_{i=1}^{K_g}(\lambda_i x^{gi}, \lambda_i y^{gi}) \in T^g$, $\lambda_i \geqq 0$, $i = 1,\cdots,K_g$ and $\sum_{i=1}^{K_g} \lambda_i = 1$. Let $\lambda_i = 1/K_g$ for all i, $\bar{x} = X/K_g$ and $\bar{y} = Y/K_g$. Then $\sum_{i=1}^{K_g}\left(\frac{1}{K_g}x^{gi}, \frac{1}{K_g}y^{gi}\right) = (\bar{x},\bar{y}) \in T^g$. I.e., each firm has the same amount of resources to produce the same amount of outputs. Total input vector is $K_g \cdot \bar{x} = X$ and total output vector is $K_g \cdot \bar{y} = Y$. Thus $\left(x^{gi} = \bar{x},\ i = 1,\cdots,K_g\right)$ is feasible for (X,Y).

Q.E.D.

The total output vector Y can be any output vector Y that can be produced by X. In particular, it can be an efficient vector. The above proposition states that equal allocation is able to produce any efficiency output vector. This justifies Li and Ng's (1995) results that the average firm can be used to find the structural efficiency. When there are heterogeneous technologies among firms but identical technologies within firms, the following Corrollary is obvious from the previous proposition.

Corollary (3) *Suppose there are G distinct firms in an industry and each of them is* *convex with an aggregate technology $\widehat{\Im}$. If firm g has K_g identical plants, then for any $(X, Y) \in \widehat{\Im}$, then there exists a feasible allocation of (X, Y) such that $x^{gi} = \bar{x}^g$ for $i = 1, \cdots, K_g$, $g = 1, \cdots, G$ and $\sum_{g=1}^{G} K_g \bar{x}^g = X$.*

This Corollary is very useful to find out the structural efficiency in Case DS, as stated in the following proposition.

Proposition (4) *Given the setting of firms and plants in the previous corollary, the structural efficiency of the industry is*

$$H(X^o, Y^o) = \max_{\theta} \left\{ \theta : (v^g, u^g) \in T^g, g = 1, \cdots, G; \sum_{g=1}^{G} (\xi_g v^g, \xi_g u^g) = (\bar{x}^o, \theta \bar{y}^o) \right\}$$

where $\xi_g = K_g / K$.

Thus to find the structural efficiency H^*, only the technologies of individual plants in each subgroup are necessary. Using previous notations, the technology of firm g is $\Im^g = \left\{ (x, y) : \sum_{i=1}^{K_g} z_{gi} y^{gi} \geqq y,\ \sum_{i=1}^{K_g} z_{gi} x^{gi} \leqq x,\ \sum_{i=1}^{K_g} z_{gi} = 1 \right\}$. The structural efficiency is

$$\begin{aligned} & H(X^o, Y^o) = \max \theta \qquad (10) \\ & \text{subject to:} \quad \sum_{i=1}^{K_g} z_{gi} y^{gi} \geqq u^g; \quad \sum_{i=1}^{K_g} z_{gi} x^{gi} \leqq v^g; \\ & \sum z_k^g = 1,\ g = 1, \cdots, G \\ & \sum_{g=1}^{G} \left(\xi_g x^g,\ \xi_g u^g \right) = (\bar{x}^o, \theta \bar{y}^o). \end{aligned}$$

4.2 Different Firm Technologies and Different Plant Technologies within Firms

This is the most general case. We have to model heterogeneity at both firm and plant levels. When we use cross-sectional data, there is only one observation for each plant, the only choice for the firm technology is the following technology set of firm g, similar to Färe and Primont (1984 and 1993),

$$T^g = \left\{ (x,y) : \sum_{i=1}^{K_g} z_{gi} y^{gi} \geqq y; \quad \sum_{i=1}^{K_g} z_{gi} x^{gi} \leqq x; \; 0 \leqq z_{gi} \leqq 1, \; i = 1, \cdots, K_g \right\} \tag{11}$$

Färe and Primont (1993) called this the Koopmans frontier of firm g with K_g plants. The aggregate technology of the industry is

$$\widehat{\mathfrak{I}} = \left\{ \begin{array}{c} (X,Y) : \sum_{i=1}^{K_g} z_{gi} y^{gi} \geqq u^g; \quad \sum_{i=1}^{K_g} z_{gi} x^{gi} \leqq v^g; \; 0 \leqq z_{gi} \leqq t_g, \\ g = 1, \cdots, G; \; i = 1, \cdots, K_g; \\ \sum_{g=1}^{G} t_g = 1; \; \sum_{g=1}^{G} (v^g, u^g) = (X, Y) \end{array} \right\} \tag{12}$$

The structural efficiency is then

$$H(X^o, Y^o) = \max_{\theta} \left\{ \begin{array}{c} \theta : \sum_{i=1}^{K_g} z_{gi} y^{gi} \geqq u^g; \quad \sum_{i=1}^{K_g} z_{gi} x^{gi} \leqq v^g; \; 0 \leqq z_{gi} \leqq t_g, \\ g = 1, \cdots, G; \; i = 1, \cdots, K_g; \\ \sum_{g=1}^{G} t_g = 1; \; \sum_{g=1}^{G} (v^g, u^g) = (X^o, \theta Y^o) \end{array} \right\} \tag{13}$$

Since the information on each firm is so limited, the following proposition is obvious.

Proposition (5) *If the aggregate technology of the industry is constructed from Koopmans frontier (11), then the industry is always structurally efficient, i.e.,* $H(X^o, Y^o) \equiv 1$ *in (13).*

Thus it is meaningless empirically to measure the structural efficiency using the Koopmans frontier. However, in the Koopmans frontier, if all observed plants are highly productive at the observed outputs, we expect that when the plants employ more inputs, they can produce more outputs. The problem is how to extend the frontier. We think that it is possible to extend the frontier by getting information from other firms. Assume that if the productivity of a firm i is higher than another firm j at the margin, then firm j's production frontier can be used to "join" firm $i's$ frontier.

Let $U' = \begin{bmatrix} U^i \\ U^j \end{bmatrix}$ and $z' = \begin{bmatrix} z^i \\ z^j \end{bmatrix}$. Let $su^i = \sum_{k=1}^{K_i} u^{ik}$ and $sx^i = \sum_{k=1}^{K_i} x^{ik}$. A technology constructed from combining the information of both firm i and j is

$$\Im^{i'} = \left\{ \begin{array}{c} (x,y) : z'Y' \geqq y, z'X' \leqq x, 0 \leqq z_{gk} \leqq 1 \\ g = i, j;\ \ k = 1,\cdots,K_g \end{array} \right\} \tag{20}$$

Firm *i* is more productive than firm j at the margin if $\max_\theta \left\{\theta : (sx^i, \theta su^i) \in \Im^{i'}\right\} = 1$. The extended frontier of firm i is formed by including all the plants of the firms that are less productive than firm i at the margin. The stuctural efficiency of the industry is then

$$H(X^o, Y^o) = \max_\theta \left\{ \theta : (v^g, u^g) \in \Im^{g'}, g = 1, \cdots, G;\ \sum_{g=1}^{G} (v^g, u^g) = (X^o, \theta Y^o) \right\} \tag{21}$$

where $\Im^{g'}$ is the extended technology set of firm *g*. In this case, we can extend the frontiers of some firms beyond our observed plants of those firms and reallocation of inputs among firms to increase total outputs is possible.

5. The Structural Efficiency of the Agricultural Sector in China

In this paper, we choose regional data of Chinese agriculture to illustrate how the methods can be applied. Many studies have examined the productivity and efficiency growth of Chinese agriculture. However, most of them utilise provincial level data for analysis (Fan, 1991, Lin, 1992, Huang and Rozelle, 1996, Kalirajan, Obwona and Zhao, 1996, Fan and Pardey, 1997). Implicit in these studies is the assumption that the provinces share the same technology. However, China is a big country with vast climatic and geographical diversity across different regions. It covers different temperature and moisture zones. Many provinces have an area bigger than that of a typical European country. Latitudinally, China stretches from 3^o50’N to 53^o31’N. The temperature zones covered by China include equatorial, tropical, subtropical, warm temperate, temperate and cool temperate. It is estimated that 32.2% of total land ares belongs to the “humid” zone, 14.5% “subhumid”, 21.7% semi-arid and 30.8% arid (see Zhao (1994) for details of China’s geographical characteristics). Assumming the production of agricultural products in different provinces to have the same technology is obviously wrong. This could lead to misleading results in efficiency analysis. One may even argue that different counties within a province have different underlying technologies. We shall compare the two results using the two modelling strategies discussed above.

5.1 Data issues

Specifically, county-level data of agricultural production in China is utilised. We use the value-added of the primary industry as the output. We include inputs used in previous studies. We have “utilization of chemical fertilisers” and “power of agricultural machinery” to represent variable and fixed capital respectively. Besides, we have agricultural sown area and agricultural labour. Furthermore, we have chosen population density as an environmental factor that can affect the production activities. In rural China, in regions where the climatic conditions and soil fertility is more favourable, population density is higher. Thus, this variable should be able to capture the heterogeneity in the production environment. We use the data for 1999, which are available in *Social and Economic Statistics of China’s Counties (Cities) 2000*. After eliminating the observations have missing data, a total of 2036 county-level units are included in our analysis. Note that the data for the units in Tibet are all missing. Thus, the county-level units in our data set covers 30 (out of 31) Chinese provinces (including autonomous regions and centrally administered municipalities). Table 1 presents the summary statistics of these variables.

To demonstrate the vast diversity in the production environment across different regions in China, we have calculated the summary statistics of county-level population density for each province (see Table 2). We can see that the mean values of county-level population density in the coastal provinces (north coastal, Yangzi River Delta and south coastal) are all higher than the mean for all the 2036 counties. In contrast, the mean values of western provinces are substantially lower than the mean of all counties. This has resulted in the large differences in the labour productivity, which is shown in Table 3.

However, as we look into the dispersion within provinces, we can see that diversity within provinces is very large. Take Guangdong as an example. It is adjacent to Hong Kong and is one of the richest provinces in China. The maximum population density is more than 20 times of the minimum. Accordingly, the maximum labor productivity among the counties is 7 times of the minimum. Economic development (including agricultural development) has been concentrated mainly in the Pearl River Delta, leaving behind the mountainous region in the northern part of the province.

Given the enormous diversity in China, the assumption that the whole country utilizes the same technology is questionable. With different environments and factor endowments, different regions are likely to have chosen different production methods as well as output mix. It is important to take into account this heterogeneity when assessing the performance of Chinese agriculture. It is more preferable to model heterogeneity at the county level than at the provincial level.

5.2 Empirical results

Table 4 reports the results of our estimations. In the upper part of the table, we examine the impact of including a fixed environmental factor. The analysis is conducted using provincial aggregates. When we do not include the "population density" as the fixed factor, the structural efficiency index is 1.62. However, once the fixed factor is added, the structural efficiency improved substantially to 1.41. This is not surprising because when the production environment is not considered, we assume that provinces with poor performance should, in theory, be able to reach the same frontier of those with good performance. This may overstate the maximum possible output of the poor provinces and thus overstate their inefficiencies.

In the lower part of Table 4, we conduct analysis with different assumptions on the technologies at the provincial and county levels. The four cases correspond to those in Section 4. Case (1) is the simplest one, estimated with the assumption all the counties and provinces have the same technology. The structural efficiency index of the whole country is 3.29, meaning that China's agricultural output should be 2.29 times more than the existing level in 1999 if all the inefficiencies are eliminated. This is rather incredible. Case (3) introduces heterogeneous technologies at the provincial level, with counties within each province sharing the same technology. This structural efficiency increases tremendously to 2.06, implying that China's agricultural output could be roughly doubled if inefficiencies are eliminated. As we argue above, however, ignoring diversities across counties is not appropriate in China. When both counties and provinces are assumed to have different technologies, as Case (4) does, the structural efficiency index dropped to 1.23. The implied 23% more of the total output can be produced with the existing input endowment. Thi number is more credible, although it indicates that Chinese agriculture has still much room to improve. For completeness, Case (2) reports the result that assumes different technologies across counties but identical technology across provinces. The structural inefficiency index is as high as 3.84. As mentioned above, the assumptions is not very meaningful.

6. Conclusion

In this paper, we have overcome the modelling of the appearance of heterogeneous firms in an industry. In particular, we discussed the possibility of different firms even they are facing the same technology and the modelling of different technologies can have more than one ways. The introduction of the extended Koopmans frontier is particularly important. It greatly enhances our understanding of the underlying production technology of a multi-plant firm with limited data.

The vast geographical heterogeneity in China provides a good testing case. Applying the methods discussed in this paper, we have confirmed that treating each county to be the same will largely exagerate the inefficiency of the agricultural sector. By adding heterogeneity to the technology, the structural efficiency turns out to be more reasonable.

We have to emphasize that whether production units are homogeneous or heterogeneous depends on the actual situation. Li and Ng (1995) and other previous studies discussed the modelling of homogeneous units. This paper introduces other possibilities of heterogeneous units. The heterogeneity may arise from uncontrollable inputs or from different technologies. The choice of the approach adopted depends on the characteristics of the empirical sample and availability of data.

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Table 1 Summary statistics of the data set

	Value-added of primary sector	Agricultural labor	Agricultural sown area	Power of agricultural machinery	Utilization of chemical fertilisers	Population density
	RMB million	10,000	1,000 hectares	10,000 kilowatt	tonnes	Pop. per sq.km
Mean	685.2	15.0	70.6	21.3	18,681.5	306.1
Stand. Dev.	578.5	11.4	51.1	22.2	18,500.2	285.0
Maximum	3,470.9	68.8	320.0	200.7	150,493.0	3,430.1
Minimum	0.6	0.1	0.1	0.1	4.0	0.1

Table 2 Population density of Chinese Counties - by provinces (Pop. per sq. km)

Province	Geographical region	Mean	Standard Dev.	Maximum	Minimum
1 Beijing	North coastal	359.8	199.2	658.2	125.2
2 Tianjin	North coastal	438.2	87.8	538.8	344.3
3 Hebei	North coastal	480.6	243.7	982.8	42.7
4 Shanxi	Middle Yellow River	217.3	171.7	995.5	38.8
5 Inner Mongolia	Middle Yellow River	81.8	241.7	2078.9	0.1
6 Liaoning	Northeastern	221.9	145.4	874.5	71.8
7 Jilin	Northeastern	123.7	79.9	288.9	28.9
8 Heilongjiang	Northeastern	93.5	69.6	305.1	3.7
9 Shanghai	Yangzi River Delta	744.2	186.8	1004.4	565.5
10 Jiangsu	Yangzi River Delta	650.1	222.8	1367.8	252.5
11 Zhejiang	Yangzi River Delta	483.3	315.7	1355.3	88.6
12 Anhui	Middle Yangzi River	428.3	218.1	1085.6	79.8
13 Fujian	South coastal	327.6	337.5	1560.8	63.7
14 Jiangxi	Middle Yangzi River	248.5	160.1	985.2	83.9
15 Shandong	North coastal	549.9	163.5	1043.8	95.7
16 Henan	Middle Yellow River	624.9	297.7	2357.7	91.4
17 Hubei	Middle Yangzi River	325.8	187.4	955.3	24.3
18 Hunan	Middle Yangzi River	314.1	160.1	908.9	86.7
19 Guangdong	South coastal	426.3	381.1	2047.1	88.5
20 Guangxi	Southwestern	193.1	122.5	683.7	40.9
21 Hainan	South coastal	204.3	79.8	325.8	76.6
22 Chongqing	Southwestern	385.6	209.6	741.4	64.9
23 Sichuan	Southwestern	330.2	296.5	1224.5	2.5
24 Guizhou	Southwestern	203.0	105.1	857.3	64.1
25 Yunnan	Southwestern	122.2	78.0	368.5	7.5
26 Shaanxi	Middle Yellow River	221.8	224.6	1086.7	16.7
27 Gansu	Northwestern	147.2	258.5	2179.8	0.2
28 Qinghai	Northwestern	43.3	59.3	193.3	0.7
29 Ningxia	Northwestern	113.7	65.5	272.5	22.4
30 Xinjiang	Northwestern	74.2	381.3	3430.1	0.1
All counties		306.1	285.0	3430.1	0.1

Table 3 Labour productivity of Chinese Counties - by provinces (Rmb/person)

Province	Geographical region	Mean	Standard Dev.	Maximum	Minimum
1 Beijing	North coastal	12,128	5,544	24,961	7,684
2 Tianjin	North coastal	8,739	2,186	11,822	6,484
3 Hebei	North coastal	6,005	4,101	23,747	1,377
4 Shanxi	Middle Yellow River	2,461	1,635	9,025	2
5 Inner Mongolia	Middle Yellow River	11,382	18,311	150,055	2,582
6 Liaoning	Northeastern	8,827	6,249	31,433	1,980
7 Jilin	Northeastern	8,604	3,544	17,189	2,470
8 Heilongjiang	Northeastern	9,546	9,636	68,875	3,347
9 Shanghai	Yangzi River Delta	9,232	2,496	11,745	7,077
10 Jiangsu	Yangzi River Delta	7,598	3,080	19,006	3,196
11 Zhejiang	Yangzi River Delta	6,682	4,422	28,649	1,917
12 Anhui	Middle Yangzi River	3,696	958	6,703	1,868
13 Fujian	South coastal	8,944	3,373	19,338	3,211
14 Jiangxi	Middle Yangzi River	4,582	1,526	9,586	1,458
15 Shandong	North coastal	6,867	8,646	84,535	2,074
16 Henan	Middle Yellow River	3,182	1,099	6,058	1,117
17 Hubei	Middle Yangzi River	6,423	2,834	14,652	2,522
18 Hunan	Middle Yangzi River	3,944	1,432	7,261	1,526
19 Guangdong	South coastal	7,930	3,459	21,636	3,208
20 Guangxi	Southwestern	4,061	1,434	8,486	1,559
21 Hainan	South coastal	10,244	3,490	17,858	5,936
22 Chongqing	Southwestern	3,000	1,130	6,007	1,461
23 Sichuan	Southwestern	3,227	1,146	6,524	1,363
24 Guizhou	Southwestern	2,194	831	6,164	1,005
25 Yunnan	Southwestern	2,666	1,317	9,943	861
26 Shaanxi	Middle Yellow River	2,828	1,463	8,483	383
27 Gansu	Northwestern	3,433	2,303	9,118	785
28 Qinghai	Northwestern	4,014	2,101	9,053	1,107
29 Ningxia	Northwestern	3,915	2,420	8,425	940
30 Xinjiang	Northwestern	10,048	7,114	34,784	2,353
All counties		5,448	5,789	150,055	12

Table 4 Estimation results

	Methodology		Structural Efficiency
A.	*Using an environmental factor to model heterogeneity*		
Case (1)	Province as the unit of analysis (without environmental factor)		1.62
Case (2)	Province as the unit of analysis (with environmental factor)		1.41
B.	*Assuming heterogeneous technologies*		
	Assumption on provinces	Assumption on counties	
Case (1)	Same technology among provinces	Same technology within provinces	3.29
Case (2)	Same technology among provinces	Different technologies within provinces	3.84
Case (3)	Different technologies among provinces	Same technology within provinces	2.06
Case (4)	Different technologies among provinces	Different technologies within provinces	1.23